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TWO MODES OF THE ANGULAR MOTION OF ARTIFICIAL EARTH SATELLITES†

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A brief survey of the dynamics of artificial satellites with passive gravity-gradient and magnetic attitude control systems is presented. As examples, the parameters of such systems are computed for two nanosatellites in which interaction with the geomagnetic field is employed, implying stringent restrictions on mass, volume and energetics. The basic functions relating the characteristics of the motion to the parameters of such satellites are presented. © 2001 Elsevier Science Ltd. All rights reserved.

The modes of angular motion of satellites in external terrestrial force fields, observed in the 1950s and 1960s, have attracted interest as regards the intensive use of small, inexpensive satellites. These early results have provided a basis for the development of passive and semi-passive attitude control systems, naturally, with resource to modern engineering achievements. This paper presents a brief survey of some research results that may be seen as a continuation of those early analyses of satellite dynamics in external terrestrial force fields and the application of those results to small satellites.

1. TWO METHODS OF ATTITUDE CONTROL

We will direct our attention to the most widespread and most readily implemented angular motions of small satellites. The most popular method of attitude control is that in which a given side of the satellite permanently faces the Earth. In this situation, the principal axes of inertia of the satellite either coincide with the axes of the orbital reference frame (triaxial gravity-gradient attitude control) or one of the axes coincides with the local vertical and the satellite moves fairly slowly about that axis (uniaxial gravity-gradient attitude control). For the most part, this method is used for remote sensing Earth satellites and communications satellites. Another mode of angular motion, used primarily on scientifically oriented satellites, is the spin of the satellite in space, one of its axes being aligned with the magnetic intensity vector **H** of the geomagnetic field (uniaxial magnetic attitude control).

Both methods – and also, incidentally, other modes of passive motion of a satellite – require, besides a *restoring* torque, which guarantees the existence and stability of an equilibrium position of the satellite relative to the selected frame of reference, a *damping* torque, which guarantees asymptotic stability of the selected mode of motion. The latter is necessary because there is almost no natural energy of dissipation of perturbed motion of the satellite relative to its centre of mass. The realization of such a torque necessitates providing the attitude control system with special damping elements.

2. GRAVITY-GRADIENT ATTITUDE CONTROL SYSTEMS

The investigation of the stability of the relative equilibrium of the Moon goes back to Lagrange. Only after two centuries did the applied aspects of this fundamental phenomenon find engineering applications. The idea of using the gravitational field to ensure a fixed attitude of the satellite relative to the Earth was proposed in 1954 by Okhotsimskii [1, 2]. A few years later, in 1959, Beletskii established conditions for the stability of an equilibrium position of a triaxial body in a circular orbit in a central gravitational field [3]. Later, he himself derived [3, 4] an equation for the oscillations of the satellite in the plane of an elliptic orbit

$$(1+e\cos\nu)\alpha''-2e\alpha'\sin\nu+\mu\sin\alpha\cos\alpha=2e\sin\nu \qquad (2.1)$$

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where $\mu = 3(A - C)/B$; A, B and C are the principal central moments of inertia of the satellite, which is spinning about its axis with moment of inertia B, the axis about which the moment of inertia is C makes an angle α with the local vertical, e is the eccentricity of the orbit, υ is the true anomaly, primes denote differentiation with respect to υ, and μ and e satisfy the inequalities $|\mu| \leqslant 4$, $0 \leqslant e < 1$. This equation has been investigated by asymptotic techniques [3, 5], and a more thorough investigation of its solutions has been achieved by numerical techniques [6]. The equation has become a classical one in spaceflight dynamics.

The next step bringing researchers closer to real attitude control systems was the choice of a damping device and of mathematical models to describe it, and an analysis of the corresponding equations of motion of the satellite taking the damping torque into account. It is the presence of a damping device that guarantees asymptotic stability of the computed motion. Of the known damping devices, hysteresis rods of magnetically soft materials were selected. In the context of the simplest hysteresis model, namely, the parallelogram model, with the dipole model used to approximate the geomagnetic field, satellite motion in the plane of a circular polar orbit was analysed [7, 8] using asymptotic methods. The conclusion was that the damping effect is proportional to the volume of the rods, and their presence generates a perturbation of the forced motions of the satellite.

In developing a passive gravity-gradient attitude control system with hysteresis rods for a real satellite, we had to refine the hysteresis model and investigate satellite dynamics for orbits of arbitrary inclination and eccentricity, and also with the expected initial conditions of angular motion. The need for a compromise between the reliability of the model and clarity of the results forced us first to use a combination of simple models and asymptotic techniques in order to determine the system parameters using the closed formulae thus obtained, and then to test system's behaviour with these parameters by numerical simulation using the refined hysteresis model.

The notation used is as follows. The orbital frame of reference $OX_1X_2X_3$ relative to which the satellite's attitude is to be controlled is defined in the traditional way. The origin is the satellite's centre of mass, the axis OX_3 is aligned with the instantaneous radius vector of the satellite's centre of mass relative to the Earth's centre, the axis OX_2 is aligned with the normal to the orbital plane and the axis OX_1 completes the frame of reference to a right-handed coordinate system. The axes of the frame of reference attached to the satellite, $Ox_1x_2x_3$, are aligned with its principal central axes of inertia and, in the case of ideal attitude, coincide with the similarly named axes of the orbital frame of reference. By a pitching (rolling yawing) rotation of the satellite we mean a rotation through an angle α about the normal to the orbital plane (through an angle γ about the velocity vector of the centre of mass and through an angle β about the local vertical). If the satellite is engaged in some composite movement – successive rotations about two or three axes – these definitions retain their physical meanings only for small angles of rotation.

We will use the parallelogram model to obtain simple relations between the parameters of the attitude control system and the dynamic characteristics of the satellite, in accordance with which the magnetic induction B_τ is expressed as a function of the projection H_τ of the external magnetic field onto the rod axis and its magnetic permeability μ by the formula

$$B_\tau = \mu\left(H_\tau - H_c \operatorname{sign}\frac{dH_\tau}{dt}\right) \tag{2.2}$$

This model probably reflects a fundamental property of hysteresis: weak dependence of the induction of the rod on the magnitude of the magnetic reversal rate, but a substantial dependence on the sign of the rate. In addition, the model is usable in both analytical and numerical work. It enables one to draw qualitative rather than quantitative conclusions. For quantitative data a numerically more accurate model will be used [9]. The first problem to be solved is the minimum necessary number of rods. Let us consider the satellite's motion in plane of polar circular orbit.

Suppose two hysteresis rods of identical characteristics are mounted on the satellite, with unit vectors $\mathbf{g}_1$ and $\mathbf{g}_2$ and with coercive force H_τ equal to zero. Considering this case within the framework of the parallelogram model, one finds that the hysteresis loop degenerates into a straight line passing through the origin, and if the total induced magnetic moment of the rods coincides with $\mathbf{H}$ at each instant of time, the geomagnetic field will not affect the satellite's motion. For identical rods with a linear magnetization characteristic, this is equivalent to the condition that the vector sum of the components of $\mathbf{H}$ along the rods must satisfy the relation

$$(\mathbf{Hg}_1)\mathbf{g}_1 + (\mathbf{Hg}_2)\mathbf{g}_2 = \mathbf{H} \tag{2.3}$$

Evaluating the scalar product of both sides of Eq. (2.3) with $\mathbf{g}_1$, we obtain $(\mathbf{Hg}_2)(\mathbf{g}_1\mathbf{g}_2) = 0$. Hence it follows that necessarily $(\mathbf{g}_1\mathbf{g}_2) = 0$, meaning that a decrease in the contribution of the magnetization of the hysteresis rods to the amplitude of steady motion is achieved by their mutual orthogonal position.

We will now assume that two mutually orthogonal rods are mounted on the satellite, and that the non-linearity in magnetization is due solely to the presence of hysteresis. Let e_1 be the projection of one of the rods onto the axis of the mean moment of inertia, in the nominal motion of the satellite directed along the tangent to the orbit. The equation of small oscillations of the satellite in the orbital plane, averaged over the natural oscillations and the argument of the latitude, has the following solution [10]

$$A_p(u) = A_p(u_0)\exp\left[-\frac{9\varepsilon H_c \nu_p}{2\pi}(u-u_0)\right] \tag{2.4}$$

where

$$\nu_p = e_1^2(1-e_1^2)[(4-3e_1^2)^{-3/2} + (3e_1^2+1)^{-3/2}] \tag{2.5}$$

A_p is the amplitude of small oscillations of the satellite, ε is the dimensionless damping coefficient of the rod and is u the argument of the latitude.

It is of interest to investigate the dependence of the coefficient υ_p on the orientation of the rods relative to the satellite's principal axes of inertia. The quantity υ_p has a maximum at $e_1^2 = 1/2$, equal to $\sqrt{2}/(5\sqrt{5}) = 0.127$. This means that damping is most effective with a "cross" of two mutually orthogonal identical rods, positioned at an angle of 45° to the principal axes of inertia of the satellite in the orbital plane.

When the rods are positioned parallel to these axes, $\upsilon_p = 0$, and in the first approximation there is no damping. Consideration of terms of higher order in the small angle α has shown [10] that in this case the equation for the amplitude of small oscillations has the form

$$A_p' = -\frac{81\varepsilon H_c \varphi^2}{16\pi}A_p - \frac{81\varepsilon H_c}{64\pi}A_p^3 \tag{2.6}$$

where φ is the angle of rotation of the rods about the satellite's principal axes of inertia. Under these conditions the angles φ and α are of the same order of magnitude. The solution of Eq. (2.6) as $\varphi \to 0$ determines the amplitude A_p as a power of the argument of the latitude

$$A_p = A_p(u_0)\left[1 + \frac{81\varepsilon H_c A_p^2(u_0)}{32\pi}(u-u_0)\right]^{-1/2} \tag{2.7}$$

Thus, positioning the rods parallel to the principal axes of inertia leads to a change in the form of the time-dependence of the amplitude. Of course, any interpretation of the above results must allow for the limited nature of the hysteresis model described by function (2.2). However, numerical investigation of satellite dynamics using both model (2.2) and the fairly adequate hysteresis model of [9] have shown good agreement with the above results.

In the case of three-dimensional motion, three mutually orthogonal rods should be mounted on the satellite. The conclusions as to the position of two rods in the plane of the axes of the mean and minor moments of inertia remain valid. The third rod, aligned with the axis of the major moment of inertia, ensures effective damping of small oscillations-rolling and yawing oscillations are damped in a short time [11] within the framework of model (2.2).

Indeed, instead of the small amplitudes A_2 and A_3 of yawing and rolling oscillations about the trivial equilibrium position, we introduce variables r and χ by formulae $A_2 = r\sin\chi$, $A_3 = r\cos\chi$. In these new variables, the equations of motion averaged over the argument of the latitude and the two phases corresponding to rolling and yawing oscillations, with a rod mounted perpendicular to the optimal "X-shape" of the two other rods, become

$$r' = -(R_2\cos^2\chi + R_3\sin^2\chi),\quad \chi' = \frac{1}{2}R_1\sin 2\chi$$

where R_2 and R_3 are functions of the inertia parameters of the satellite, which are proportional to the dimensionless damping coefficient ε, the coercive force H_c of the rod material and the sine of the angle of inclination of the orbit. The condition for asymptotic stability of the position $r = 0$ is $R_2 > 0$, $R_3 > 0$. The response time of the system is determined by the quantity $\min(R_2, R_3)$. The rate of damping of oscillations in the neighbourhood of the position $r = 0$ is proportional to the product $\varepsilon H_c \sin i$, implying the greater efficacy of the configuration just described in near-polar orbits.

When the results of the theoretical analysis were used to develop an actual gravity-gradient attitude control system for the REFLECTOR nanosatellite of mass 6 kg, it was necessary to tackle additional problems relating to the simulation of satellite dynamics with the refined hysteresis model [9], taking into account mutual demagnetization of the rods, resonance relations between the natural frequencies and the frequencies of variations in the external torques, the effect of magnetizable elements of the satellites on its dynamics and the state of the rods.†

The criterion chosen to govern the final selection of moments of inertia in the REFLECTOR satellite was the amplitude of yawing oscillations. A forced solution, under the action of a perturbing torque due to residual magnetization in the rods, was obtained from the equations of the oscillations, linearized in the neighbourhood of the trivial position of the satellite in the orbital frame of reference. The hysteresis model selected was the Rayleigh model, which allows for non-linear magnetization of the rod, since in the parallelogram model it has been shown that when three identical mutually orthogonal rods are installed, the effect of linear magnetization is eliminated. As the derivation of the solution is quite cumbersome, for simplicity we will present only the expression for the yaw angle, which is the most crucial one for the functioning of the system

$$\beta = \frac{\mu_0 V_b \alpha_R H_0^3}{\omega_0^2} \sin i \cos i \frac{2(B-C)\cos i + A(\cos i - 3\sin i)}{3(B-C)(A-B+C)} \cos u \tag{2.8}$$

where μ_0 is the magnetic constant, V_b is the volume of the rods, H_0 is the modulus of **H** at the equator and α_R is the Rayleigh constant. Since the values of the equatorial moments of inertia are close together and their difference is close to the axial moment of inertia of the satellite, the element in this solution which exerts the greatest influence on the amplitude of the oscillations of the angle β is the factor in parentheses $(A - B + C)$. It follows from solution (2.8) that placing two additional loads along the Ox_1 axis does not cause an increase in the value of $(B - A - C)$, which might reduce the effective yawing torque. Indeed, as the mass of the additional loads on the Ox_1 axis is increased, the increment to the moment B is exactly balanced by the increase in the moments C and A. Additional loads must also be placed on the satellite axis Ox_2, which, in the equilibrium position, is aligned with the normal to the orbital plane.

The criteria governing the amplitude of yawing oscillations were minimization and sufficient distance from the resonances defined by

$$\omega_1 = 2\omega_3, \quad \omega_1 = \omega_2 - \omega_3, \quad \omega_1 = \omega_2 + \omega_3 \tag{2.9}$$

where ω_1 is the frequency of pitching oscillations, and ω_2 and ω_3 are the frequencies of spatial oscillations; on this basis, analysis of the amplitude of yawing oscillations enabled a rational distribution to be made of additional loads along the Ox_1 and Ox_2 axes, provided the structural constraints on the total mass of the loads were observed. In the final design, eighteen rods were installed on the satellite, with a total volume of 0.25 cm^3. After the additional loads had been installed, the principal central moments of inertia of the satellite were $A = 1.633$, $B = 1.795$ and $C = 0.256$ kg m^2. The initial condition which exerted the greatest influence on the duration of the transient was the angular velocity. The initial values of the components of the angular velocity of the satellite were estimated at approximately 3° per second. The transient lasted approximately 1 month. The angles θ_k $(k = 1, 2, 3)$ between similarly designated axes of the orbital and satellite frames of reference, as functions of the argument of the latitude u, describe the almost steady motion of the satellite during that time. The effect of the Earth's diurnal rotation on the motion of the satellite was quite marked. The geomagnetic field was approximated by the field of a dipole situated at the centre of the Earth, at an angle of 11.5° to its axis of rotation. In accordance with the results of available estimates, the aerodynamics and light pressure effects were ignored. The period of slow evolutionary changes in the motion characteristics was close to one day – the period of oscillations of the geomagnetic dipole with respect to the orbital plane. The major influence

†OVCHINNIKOV, M. Yu., PEN'KOV, V. I. and MIRER, S. A. Passive gravitational attitude control system of the REFLECTOR nanosatellite. Preprint No. 30 M. V. Keldysh, Inst. Prikl. Mat. Ross. Akad. Nauk Moscow, 1999.

turned out to be on the yawing motion of the satellite. The angle θ_3 between the longitudinal axis and the local vertical varied insignificantly over a period of 24 hours.

Based on the results of a numerical simulation, it should be made that for rods which actually move within the orbital plane after the transient is completed, the magnetization loop is almost parallelogram-shaped, while for rods almost perpendicular to the orbital plane the magnetic reversal regime is close to that described by the Rayleigh model.

3. MAGNETIC ATTITUDE CONTROL SYSTEMS

The precessional motion of a satellite with a permanent magnet has been considered in the context of the averaged direct dipole model. According to this model, the geomagnetic field intensity vector does not change in magnitude and moves uniformly over a conical surface in a translating frame of reference with origin at the satellite's centre of mass. The dynamics of the first satellite with a passive magnetic attitude control system. Transit-1B, was described in [13]. The now classical equation of motion for a satellite with a permanent magnet in the plane of a polar orbit about the vector **H**, first published by Beletskii [4], is

$$\alpha'' + \eta\sqrt{1+3\sin^2 u}\,\sin\alpha = \frac{6\sin 2u}{(1+3\sin^2 u)^2} \tag{3.1}$$

where η is the dimensionless magnetic parameter of the satellite, which is proportional to the magnitude of the magnetic restoring torque $m_s H_0$ and inversely proportional to the quantity $B\omega_0^2$.

Using this equation as an example, the following factors influencing the satellite's motion were identified. First, the fact that the equation has a non-zero right-hand side implies that exact orientation of the satellite along the vector **H** is impossible. Second, after expanding the right-hand side in a Fourier series in harmonics, with frequencies which are multiples of twice the orbital frequency ω_0 of the satellite's centre of mass, one becomes aware of the presence of natural frequencies of small oscillations of the satellite about the vector **H**, and if these are identical with the frequencies of the harmonics in the Fourier series, resonance may occur. Third, the coefficient of $\sin\alpha$ is a periodic function of time, which gives rise to variation of the amplitude curve, which is slow compared with the period of natural oscillations of a satellite with a sufficiently strong magnet, over the period of revolution of the satellite in orbit about the Earth. Finally, because of the periodic dependence of this coefficient, parametric resonance may occur in the relative motion of the satellite.

Equation (3.1) was investigated fairly thoroughly by the method of harmonic balance in publications by Beletskii and Khentov, and it was generalized in the monograph [7]. In particular, several first resonance values of the parameter η have been found.

If a passive magnetic attitude control system is to be developed for a specific satellite, it is naturally necessary to investigate its dynamics in detail. Equations of motion have been derived for a satellite with a magnet, assuming the direct dipole model to approximate the geomagnetic field.† The assumption that the satellite is axisymmetric made it possible to reduce the order of the system of equations to four. Consideration of a particular solution of the equations – motion of the satellite's axes of symmetry in the plane of a polar orbit – and use of the Sturm–Liouville theorem, for a sufficiently strong magnet, yielded asymptotic formulae for the points of bifurcation of periodic solutions. Periodic solutions are indeed those obtained as computed motions of the satellite.

As indicated previously, one source of bifurcation of solutions is resonance between the natural frequencies of the satellite and the frequencies of perturbations arising from non-uniform rotation of the vector **H** due to the satellite's orbital motion. Motions bifurcate in the orbital plane, splitting off three-dimensional motions. The latter become valid three-dimensional motions if the satellite is spinning about an axis and the orbit is symmetric or non-polar. Formulae of identical form define the bifurcation values of the parameter η for bifurcation points of both types:

$$\eta = p_1 k^2 + p_2 + O(1/k) \tag{3.2}$$

where k is the order number of the bifurcation points and p_1 and p_2 are certain constants, different but constant for each type of bifurcation. For orbits of arbitrary inclination, a formula structurally analogous

†SARYCHEV, V. A., SAZONOV, V. V. and OVCHINNIKOV, M. Yu., Periodic oscillations of a satellite about the centre of mass under the action of a magnetic torque. Preprint No. 182. M. V. Keldysh Inst. Prikl. Mat., Akad. Nauk SSSR. Moscow, 1982.

to (3.2) has been obtained, with p_1 and p_2 dependent on the inclination of the orbit [14]. Figure 1 presents numerically computed bifurcation curves for symmetric solutions in the (η, i) plane. The solid curves correspond to small-amplitude solutions. The dashed curves represent bifurcation curves of solutions with fairly large amplitudes. The bifurcation points of plane solutions and points at which three-dimensional solutions branch off are denoted by P_1, R_1 and Q_2, respectively. Results of a detailed investigation of the periodic solutions of satellite with a permanent magnet are presented in [15]. The main results of the investigations are: the determination of the values of the magnetic parameter of the satellite which are "dangerous" from the standpoint of increasing amplitude of oscillations about the vector **H**, estimates of the amplitude of oscillations as a function of the values of the magnetic parameter and the inclination of the orbit and an analysis of the necessary conditions for these oscillations to be stable.

The next step is the choice of the parameters and of the number and position of hysteresis rods in the body of the satellite. For plane motion in the context of the parallelogram model, a way to obtain estimating characteristics of the transient has been indicated [8]. Mention should be made of [16], in which a general expression was obtained for the amplitude of the oscillations of the satellite in the plane of a polar orbit for fairly general assumptions concerning the shape of the hysteresis loop. To study the evolution of the three-dimensional motions of a satellite with rods in an orbit of arbitrary inclination, an analysis of such motions was necessary. This was first done using asymptotic techniques, on the assumption that the influence of the rods while the satellite is orbiting was insignificant and that the angle θ by which the axis of symmetry of the satellite deviated from the vector **H** was small, in the context of the Rayleigh model [17]. The following expression was obtained for the angle θ

$$\theta(u) = \frac{\theta(u_0)}{(1+3\sin^2 i \sin^2 u)^{5/8}} \exp\left(-\frac{2}{3\pi}\bar{\varepsilon}\int_{u_0}^{u}(1+3\sin^2 i \sin^2 u)^{5/4} du \right)$$
$$S_h = \frac{16}{3}\alpha_R H_{rm}^3, \quad \bar{\varepsilon} = \frac{\alpha_R V_b H_0^2}{m_s} \tag{3.3}$$

(in the Rayleigh model the area S_h of the hysteresis loop is expressed in terms of the Rayleigh constant α_R and the amplitude H_{rm} of the magnetic reversal field). For small H_{rm}, the quantity α_r remains constant. One can therefore determine S_h from experimental data and compute α_R. Having experimental data on the loops, one can obtain a qualitative description of the transient using the previous formulae. Incidentally, the last formula shows that the coefficient $\bar{\varepsilon}$ is inversely proportional to the modulus m_s of the dipole moment of the permanent magnet.

The next step was as numerical investigation of the transient and steady motions of the satellite. We will consider this using the example of the MUNIN nanosatellite [18], which is equipped with a permanent magnet and a number of hysteresis rods. Based on results obtained by analysing the periodic solutions of the satellite, the dipole moment m_s of the permanent magnet was taken to be equal to 0.3 A m^2. The total volume of the six rods, each of length 15.5 cm, was about 1 cm^3. An example of the steady motion, represented by the angle θ as a function of the argument of latitude, in two loops around the Earth, is shown in Fig. 2, which was obtained using the IGRF-95 model of the geomagnetic field and the refined hysteresis model. Figure 3 shows the magnetization of the rods along an axis perpendi-

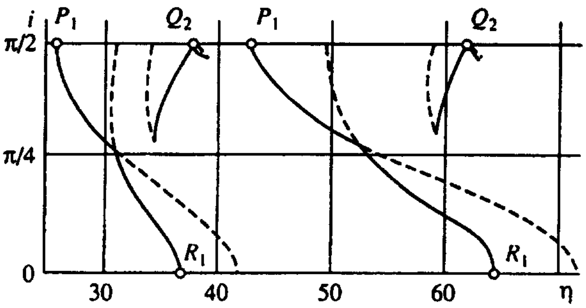

Fig. 1

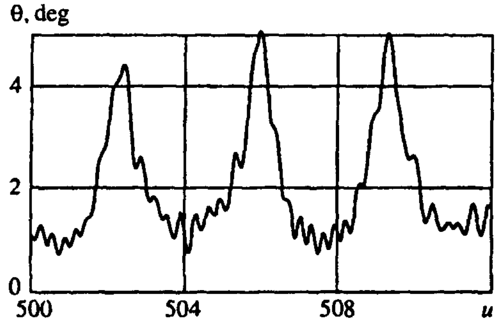

Fig. 2

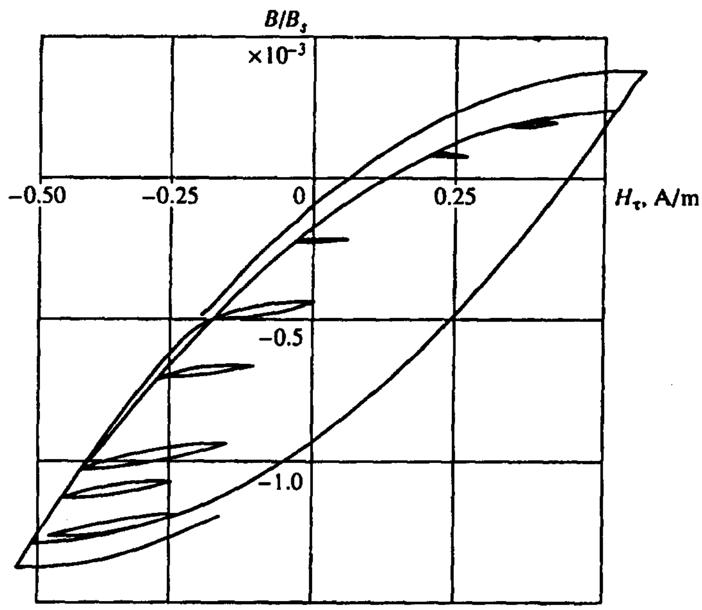

Fig. 3

cular to the orientation axis, divided by the saturation induction, which is equal to 0.74 T, plotted as a function of the magnitude of the projections of the geomagnetic field intensity vector onto the rods, for the duration of one revolution of the satellite in its orbit. The partial loops correspond to the magnetic reversal of a rod in forced oscillations at frequencies close to the natural frequency.

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